# CRYO-ELECTRONIC & NOISE

# DÉTECTION DE RAYONNEMENT À TRÈS BASSE TEMPÉRATURE

Damien PRÊLE - APC DRTBT2024 - Mars 2024 https://drtbt.neel.cnrs.fr

# Ultra sensitive cryogenic detectors

- Bolometers / Calorimeters
- Josephson junction based detectors like SQUID (Superconducting QUantum Interference Device) and STJ (Superconducting Tunnel Junctions)
- **KID** (Kinetic Inductance Detector)
- SNSPD (Superconducting Nanowire Single-Photon Detector)

Front-end cryogenic **readout** requirement Basic function of a front-end readout → **AMPLIFICATION** 

CRYOGENIC environment allows to achieve extremely low noise performances ⇒ NOISE of the cryogenic readout chain is an essential point Anyway, Cryogenic T is not a "magic" solution to reduce noise

# Amplification = increase in amplitude

- Power: active devices are required for power applications
- Voltage or Current: Current or voltage amplification can be done at equal power using transformers or resonant circuit



■ A *buffer* (common emiter) voltage gain = 1, provides a current gain (high  $Z_{in}$  and low  $Z_{out}$ ) leading to power gain  $G_P = G_U G_I$  avec  $G_U \rightarrow$  and  $G_I \nearrow$ 

 $^{137}$  A opamp  $(Z_{out}\approx 50\Omega)$  amplifying a voltage signal of a 50 Source, provides also a power gain

IS A MOS with an isolated grid, provids a power gain  $\rightarrow \infty$ !

# SNR degradation & readout detection chain

The Front End amplifier stage of a tiny signal is at the same times :

- The require function to prevent noise/parasite contaminations,
- The **main source of noise** of the readout chain.

It is important to amplify as early as possible in detection chain and avoid any signal attenuation. Once amplified, the amplitude of the signal becomes large compared to the amplitude of the following stages noise.

# Thermal noise Identify by John Bertrand Johnson in 1926

#### thermal agitation of the charge carriers inside an electrical conductor

50

#### NATURE

#### [JANUARY 8, 1927

#### Thermal Agitation of Electricity in Conductors.

ORDINARY electric conductors are sources of spontaneous fluctuations of voltage which can be measured with sufficiently sensitive instruments. This property of conductors appears to be the result of thermal agitation of the electric charges in the material of the conductor.

The effect has been observed and measured for various conductors, in the form of resistance units, by means of a vacuum tube amplifier terminated in a thermocouple. It manifests itself as a part of the phenomenon which is commonly called 'tube noise.' The part of the effect originating in the resistance gives rise to a mean square voltage fluctuation  $V^*$ which is proportional to the value R of that resistance. The ratio  $V^2/R$  is independent of the nature or shape of the conductor, being the same for resistances of metal wire, graphite, thin metallic films, films of drawing ink, and strong or weak electrolytes. It does, however, depend on temperature and is proportional to the absolute temperature of the resistance. This dependence on temperature demonstrates that the component of the noise which is proportional to R comes from the conductor and not from the vacuum tube.

No. 2984, Vol. 119]

A similar phenomenon appears to have been observed and correctly interpreted in connexion with a current sensitive instrument, the string galvanometer (W. Einthoven, W. F. Einthoven, W. van der Horst, and H. Hirschfeld, *Physica*, 5, 358-360, No. 11/12, 1925). What is being measured in these cases is the effect upon the measuring device of continual shock excitation resulting from the random interchange of thermal energy and energy of electric potential or current in the conductor. Since the effect is the same for different conductors, it is evidently not dependent on the specific mechanism of conduction.

The amount and character of the observed noise depend upon the frequency-characteristic of the amplifier, as would be expected from experience with the small-shot effect. The apparent input power originating in the resistance is of the order 10<sup>-18</sup> watt at room temperature. The corresponding output power is proportional to the area under the graph of power amplification-frequency, at least in the range of audio frequencies. The magnitude of the 'initial noise,' when the quietest tubes are used without input resistance, is about the same as that produced by a resistance of 5000 ohms at room temperature in the input circuit. For the technique of amplification, therefore, the effect means that the limit to the smallness of voltage which can be usefully amplified is often set, not by the vacuum tube, but by the very matter of which electrical circuits are J. B. Johnson. built.

Bell Telephone Laboratories, Inc.,

New York, N.Y., Nov. 17.

# Thermal noise Measured by John B. Johnson in 1926

## Voltage noise $\propto \sqrt{R}$ independent of the nature of the resistance





Thermal Agitation of Electricity J. B. Johnson, PHYSICAL REVIEW, July, 1928

John Bertrand Johnson (1887-1970)

# Thermal noise Measured by John B. Johnson in 1926

#### DSP $\propto$ à T Independently to the R value





Thermal Agitation of Electricity J. B. Johnson, PHYSICAL REVIEW, July, 1928

John Bertrand Johnson (1887-1970)

# Thermal noise Interpreted by Harry Nyquist in 1928

JULY, 1928

PHYSICAL REVIEW

VOLUME 32

#### THERMAL AGITATION OF ELECTRIC CHARGE IN CONDUCTORS\*

By H. Nyquist

ABSTRACT

The electromotive force due to thermal agitation in conductors is calculated by means of principles in thermodynamics and statistical mechanics.





# Quantum limitation (@ high *f* and/or verry low *T*)

Generalization including a quantum approach (similar to black body radiation) :

Electric energy  $\rightarrow$  multiply by hf DSP thermal noise :

$$S = \frac{hf}{e^{\frac{hf}{k_BT}} - 1}$$
  
=  $\frac{hf}{\frac{hf}{k_BT} + \frac{1}{2!}(\frac{hf}{k_BT})^2 + \frac{1}{3!}(\frac{hf}{k_BT})^3 + \dots}$ 

$$\approx k_B T$$
 only if  $k_B T >> hf$ 

#### ☞ @ 300 K white up to 6 THz ☞ @ 300 mK "only" up to 6 GHz !

# DSP de bruit thermique d'une resistance : $S = \frac{hf}{e^{\frac{hf}{k_BT}} - 1}$



# Johnson noise cut off take in consideration for Johnson thermometry

At the very lowest temperatures, Nyquist's law may not be a sufficient approximation of the fluctuation dissipation relation (1). The relative error in Nyquist's law is given by the series expansion of the Planck factor (see (2))

$$\frac{(hf/kT)}{\exp(hf/kT)-1} = 1 - \frac{hf}{2kT} + \dots \approx 1 - 2.4 \times 10^{-11} \frac{f}{T}, \quad (23)$$

so that a JNT using Nyquist's law at temperatures near 1 mK and with an average operating frequency of 100 kHz is accurate to about 0.24%.

J. F. Qu et al. Johnson Noise Thermometry

# Available/Measurable thermal noise power $k_B T \Delta f \rightarrow$ Puissance maximale disponible

To measure  $P = k_B T \Delta f$ , a matching impedance is required  $(R_1 = R_2 = R)$  -> Optimal power transmission



# Voltage and Current noise spectral density

# $e_n = \sqrt{4k_B T R} [V/\sqrt{Hz}]$



$$i_n = \sqrt{\frac{4k_BT}{R}} \left[ A/\sqrt{Hz} \right]$$



# Equivalent noise resistance and noise temperature

Assuming a white noise spectral density  $S_v$  and  $S_i$ ; and a "source/detector" impedance  $R_s$  we modeled as a :

Noise equivalent resistance: Resistance producing the same Johnson noise than the S<sub>v</sub> and S<sub>i</sub> contribution (define at which temperature).

$$R_{eq} = \frac{S_v + R_s^2 S_i}{4k_B T_0}$$

► Noise equivalent temperature : at which temperature a  $R_0$ (often  $R_0 = R_S = 50\Omega$ ) produces the same noise as the  $S_v$  and  $S_i$  contributions

$$T_{eq} = \frac{S_v + R_s^2 S_i}{4k_B R_0}$$

# Noise in electronic devices

#### Johnson noise = Real resistances

- ✓ The Johnson noise is only associated to the ℜ impedance
- ➤ Imaginary parts (L or C) and Dynamic impedances or active charges  $(h_{11}, 1/g_m, R_{DS} ...) \rightarrow$  do not produces Johnson noise (except maybe the MOS channel noise in Strong inversion  $\frac{2}{3}4k_BTg_m$  where the resistance of the channel is real)

#### But sometimes people misuse the term by referring to ...

- "Capacitive" noise  $\frac{k_B T}{C}$
- "Thermal noise of a bipolar transistor"  $\frac{4k_BT}{2g_m}$

# Voltage noise $k_B T/C$ (noise integrated f[0-> $\infty$ ])

#### integrated noise of a resistor filtred by a capacitor



#### ⇔but T is the resistor temperature

# Charge noise $k_B TC$

$$Q = CV$$
  
 $v_{eff} = \sqrt{\frac{k_B T}{C}} \longrightarrow Q_n = \sqrt{k_B TC}$ 

#### Charge and voltage noise in capacitors

C	$\frac{\sqrt{k_BTC}}{q}$	$\sqrt{k_B T/C}$
$1 \mu\text{F}$	400000 e <sup>-</sup>	64 <i>n</i> V
1 nF	13000 e <sup>-</sup>	2 µV
1 pF	$400 e^{-}$	64 µV
1 fF	13 e <sup>-</sup>	2 <i>m</i> V
1 aF	0,4 <i>e</i> <sup>-</sup>	64 <i>m</i> V

discrete nature of electric charge ; visible in the fast transit of  $e^-$  in a potential depleted zone / junction<sub>(Poisson distribution)</sub>



a "Shot !" occurs every electron passing throug the junction depleted zone

Current = discrete transfert of charges q at arbitrary time  $t_n$ 

# Shot noise in a junction

- Shot noise  $\rightarrow$  fast transit
- Time correlation crossing the device
  - neglectable recombinations (nb  $e^-$  emitted = received) neglectable latice interaction / delay, large relaxation limes (average time between "collitions")  $\tau_r$

Transit time crossing the depleted zone  $\tau_t$ :



 $\tau_t \ll \tau_r$ 

*shot noise* discovered in vacum tube by Walter Schottky in 1918



# "Schottky expression" Current spectral density $[A^2/Hz]$

$$S_i = 2qI$$

with I the DC current crossing the junction

#### White noise ∀ f and T RMS noise over 1 Hz

$$i_{eff}|_{\Delta f=1Hz}=\sqrt{S_i}=\sqrt{2qI}$$

N.A.: assuming a diode bias with I = 1 mA:

$$\sqrt{S_i} = \sqrt{2 \times 1.6 \ 10^{-19} \times 10^{-3}} = 18 \ pA/\sqrt{Hz}$$

 $q\approx 1,6\times 10^{-19}$  [C] ou [A.s] est la valeur absolue de la charge de l' $e^-$ 

# Shot noise in high frequency

# Transit time $e^-$ through a junction $\tau_t$

$$q\delta(t-t_n) \stackrel{i}{ - \tau} \stackrel{--\tau}{ - \tau} \stackrel{-}{ - \tau} \stackrel{q}{ - \tau} \stackrel{i}{ - \tau} \stackrel{\tau_t}{ - \tau} \stackrel{t}{ - \tau} \stackrel{t}{$$

## Shot noise spectral density in HF

$$S_{i} = 2qI \left[\frac{\sin(\pi f\tau)}{\pi f\tau}\right]^{2}$$

$$A.N. \tau_{t} = 10ps: = \frac{\pi}{2}^{2ql/10}$$

$$\frac{1}{\tau} = 100GHz! = 2ql/100$$

$$0.1/tau = \frac{1}{tau}$$

$$1/tau = 10/tau$$

# Small signal noise model

# diode equivalent noise small signal schematic



$$r_d = \frac{k_B T}{q I_D}$$

$$S_i = 2qI_D$$

voltage noise  $S_i = 2qI_D \left(\frac{k_BT}{qI_D}\right)^2 = 2\frac{(k_BT)^2}{qI_D} \dots 4k_BT\frac{r_d}{2}$ is finally, shot noise is depending on temperature ... but it is not means that it is a Johnson noise !

# Non-Fondamental noise

Excess noises  $\neq$  Johnson & Shot)

Non/Less predictable noise, not well understood

Shot noise may be include in excess noise, because it disappear with no bias

- Avalanche -> M factor on the shot noise
- Generation recombinaison (GR ou burst noise)
  - Telegraphic noise
  - 1/f (Flicker noise)

# Noise in commercial SMD with current



Noise of surface-mount devices (SMD) with  $100 \Omega$  up to a max. power dissipation of 1 W (U=10V)"Resistor Current Noise Measurements" - Frank Seifert

# Low frequency noise $\rightarrow$ traps

The low frequency noise is associated to traps

### Generation recombinaison

#### Arbitrary number of carriers under electric field:

∕ generation :

- $e^-$  trap  $\longrightarrow$  bande de conduction
- *trou* trap  $\rightarrow$  bande de valence

 $\searrow$  recombinaisons :

 $e^-$  bande de conduction  $\longrightarrow$  trap *trou* bande de valence  $\longrightarrow$  trap

# Generation-recombinaisons - GR noise

#### drift current fluctuation

# Arbitrary rate of generation recombinaisons in traps (defects, doping, stats of surface)



Generation-recombinations  $\rightarrow$  A single G-R process dominats  $\tau_r$  characteristics of an exponential "relaxation"

# Different relaxation times superposition

multiple generations - recombinaison process

# Telegraphic noise 2 levels system or more: multiple time constants $\tau_r$ $S_{2level} \propto \frac{4(\Delta I)^2}{(\tau_1 + \tau_2) \left[ \left( \frac{1}{\tau_1} + \frac{1}{\tau_2} \right)^2 + (2\pi f)^2 \right]}$



## 1/f - flicker noise

Large number of generations-recombinaisons processes  $\rightarrow$  large number of time constants  $\tau_i$ 

# Flicker noise

► Large number of G-R:  $e^-$  number fluctuation ; due to defect, ion and any traps (recombinaison center - present in surface)  $\rightarrow$  Large number of Lorentzian spectrums ( $\neq f_c$ )



Mobility fluctuation leads also to: "volume" noise

# Low frequency noise bellow 0.1 Hz ... and thermal drift

$$f < 10^{-2} Hz \rightarrow$$
 Thermal fluctuations

Measured noise down to about  $10^{-5}Hz \equiv 1$  day :

- The drifts are indistinguishable from the low frequency noise
- The Johnson noise is not anymore white ... because T is not anymore a constante

# Allan variance

✓ integration times  $\tau$ ≠ ✓ SNR





# Empirical modelization of the 1/f noise



# Noise syntesis

Nom	Forme	DSA $S_i$	Source
Johnson	white	$4k_BT/R$	Real resistance
shot	white	2qI	Junction
flicker	$1/f^{\gamma}$	$\frac{K}{A} \times \frac{I^{\alpha}}{f^{\gamma}}$	Semiconductor

### **Possible mitigations**

- ► Johnson : Resistance, Température
- shot : Current
- 1/f: Current density size, technology/cleanliness

# Noise Factor and Noise refer to the input (RTI)



Assuming a source resitance  $R_S$  and a voltage amplifier, the noise factor NF :

$$NF = 10\log \frac{\frac{V_{in}^2}{4k_B T R_S}}{\frac{(GV_{in})^2}{G^2(4k_B T R_S + e_n^2 + R_S^2 i_n^2)}} = 10\log \frac{4k_B T R_S + e_n^2 + R_S^2 i_n^2}{4k_B T R_S}$$

For  $e_n^2$  and  $i_n^2$  here the spectral density and G=  $\frac{V_{OUt}}{V_{in}}$  the voltage noise

# Input noise vs source resistance

$$NF = 10\log \frac{4k_B T R_S + e_n^2 + R_S^2 i_n^2}{4k_B T R_S}$$

Example: input noise vs optimal noise resistance of an OP27



When total noise is close the Johnson noise -> optimal source resistance (minimum NF)

# Noise index vs source resistance



#### Analog Devices operational amplifier noise

- For a **low impedance** source -> AD797;

- For a **high impédance** source -> AD745.
- NOISE INDEX approaches 1 (0dB) for **Optimal impedance**

# Optimal impedance $R_{S_{opt}}$

$$R_{S_{opt}}$$
 : source impedance  $R_S$  : NOISE INDEX

$$F_{min} \to \frac{\partial F}{\partial R_S} = 0 \Big|_{R_S = R_{Sopt}}$$

$$\frac{\partial \left(1 + \frac{R_{S_{opt}}^2 i_n^2 + e_n^2}{4k_B T R_{S_{opt}}}\right)}{\partial R_{S_{opt}}} = 0$$

$$\frac{i_n^2}{4k_BT} - \frac{e_n^2}{4k_BTR_{S_{opt}}^2} = 0 \rightarrow \boxed{R_{S_{opt}} = \frac{e_n}{i_n}}$$
# Aop Analog Devices en fonction de $R_{S_{opt}}$

Low Noise Amp. Selection Guide for Optimal Noise Perf.



# Decreasing noise by putting Aop in //



Quadratic sum of independant noise (multipliés par  $\sqrt{N}$ ); Signal (same for all amplifier) is multiply by N.

The first non inverter stages provide N gains =  $101 (1 + 2k\Omega/20\Omega)$ ; Last stage sum (sommater-inverter):

Voltage gain=-1010

#### Cryogenic electronic devices

- 1. Field-effect Transistors FET
  - standard MOS & JFET
  - ► Hetero-junction FET *ie* HEMT
- 2. Bipolar transistors
  - Bipolar Junction Transistor BJT
  - Hetero-junction Bipolar Transistor HBT
- 3. Superconductor devices
  - Superconducting QUantum Interference Devices SQUIDs

# Amplifications and figures of merit

ignal amplification			
Input signal	⇒ Gain	$\Rightarrow$ Output signal	
AMPLIFICATION	UNIT	EXEMPLES	
voltage	[V/V] transformer; op. amplifier		
current	[A/A]	transfo.; <b>bipolar trans.</b> ( $\beta$ )	
transimpedance	[V/A]; [Ω]	resistor, SQUID + input coil	
transconductance	[A/V]; <b>[S</b> ]	transistor $(g_m)$	

## Voltage/Current sources and "matching" impedance



voltage amplifier example

AMPLIFICATION	INPUT	OUTPUT
voltage gain	$Z_{in} > Z_S$	$Z_L > Z_{out}$
current gain	$Z_{in} < Z_S$	$Z_L < Z_{out}$
transimpedance	$Z_{in} < Z_S$	$Z_L > Z_{out}$
transconductance	$Z_{in} > Z_S$	$Z_L < Z_{out}$

#### Solid-state physics and semiconductors



#### Solid-state physics and semiconductors (carriers density as function of T)



#### Solid-state physics and semiconductors (carriers density as function of T)



#### Solid-state physics and semiconductors (carriers density as function of T)



## Field Effect Transistor - FET

# Field effect transistor uses **ELECTRIC FIELD** to control the output current

Different ways to isolate the grid

1. INSULATOR (as SiO2)

 $\rightarrow$  MOSFET (Metal Oxide Semiconductor FET)

#### 2. **Depleted region** of a reverse biased **pn JUNCTION**

 $\rightarrow$  JFET (Junction FET)

3. Depleted wide band-gap of an HETEROSTRUCTURE (as

GaAs/AlGaAs)

 $\rightarrow HEMT \quad (\text{High Electron Mobility Transistor})$ 

#### nMOS nodes & topology

#### nJFET nodes & topology







#### Parameters :

transconductance

 $g_m = \frac{\partial I_D}{\partial V_{GS}}$ 

- capacitive input impedance  $\rightarrow$  close to  $\infty$  at low freq.
- current gain not defined  $\rightarrow Z_{IN}$  too large
- output impedance depends on the circuit (what the output is)

## MOS and JFET transconductance

#### 2 different operation modes for amplification

1. weak-inversion for low consumption;  $I_D = I_{D0} \exp \frac{V_{GS} - V_{th}}{\eta V_T} \Rightarrow g_m = \frac{I_D}{\eta V_T}$   $I_{D0} = I_D \text{ and } V_{GS} = V_{th}, \eta = 1 + \frac{C_D}{C_{ox}} \text{ at}$   $V_T = \frac{k_B T}{q}$ 2. the active mode for low-noise analog amplifier

Linear low noise amplification  $\rightarrow$  *pinch-off* and **active mode** (saturation)

$$I_D(V_{DS}) \approx \kappa (V_{GS} - V_{th})^2 \text{ with } \kappa = \begin{cases} \frac{\mu C_{ox}}{2} \frac{W}{L} & \text{for MOS} \\ \frac{I_{DSS}}{V_{th}} & \text{for JFET} \end{cases}$$

$$|g_m| = |\frac{\partial I_D}{\partial V_{GS}}| \approx 2\kappa (V_{GS} - V_{th}) \propto \sqrt{I_D}$$

$$\mu(T) \propto T^{-\alpha} \rightarrow \mu \nearrow \text{ at low temperature} \Rightarrow g_m \propto \frac{\sqrt{I_D}}{T^{\alpha}}$$

#### Cryogenic measurement of MOS transconductance



### MOS output characteristic and kink effect



At high  $V_D$ , e<sup>-</sup>-hole pairs created by impact ionization mechanism. • e<sup>-</sup>  $\rightarrow$  drain

► holes stay in the freezed-out bulk (increasing the bulk potential)  $\Rightarrow$  add a "potential control" in addition to  $V_{GS}$ 

Kink effect is stronger in nMOS as compared to pMOS Solution : adding many bulk contact around the MOS

## MOS transistor design for cryo. and space applications To mitigate the kink effect, all MOS transistors are fully surrounded by substrat contacts (to really fix the potential) :



Exemple of inverter in AMS 0.35 Technology

Moreover, ELT (Edge-Less Transistors) suppress the effect of the charges trapped in "bird's beak" of standard MOS design (LOCal Oxidation of Silicon)





#### Latch-up mitigation at cryogenic temperatures

- Gard ring used to reduce kink effect allows to reduce too the Latch-up occurrence
- At low temperatures, freeze-out dramatically reduce BJT (Bipolar Junction Transistor) current gain



#### MOS transistor Flicker evolution with T



measurements 28nm technology (a) nMOS and (b) pMOS at 300 and 4 K for Vov = 0 V and Vds = 20 mV. Notice that the noise amplitude at 4 K is higher than the one at 300 K Ruben Asanovski et al. Understanding the

Excess 1/f Noise in MOSFETs at Cryogenic Temperatures - 2023

#### 1/f noise excess at cryogenic temperature

- band tail states start to behave as traps at cryogenic temperatures
- related to the temperature dependence of the hopping mechanism in band tail states
- -plus dielectric traps



The trapping dynamics :

-> decrease in temperature reduce the number of traps thermally accessible, resulting in a noise spectrum originating from a few  $\tau$ only prele@apc.in2p3.fr 52 / 86 DRTBT 2024 - Aussois - du 24 au 29 mars 2024

## Cryogenic measurement of JFET transconductance

$$\begin{split} I_D(V_{DS}) &\approx I_{DSS}(1 - \frac{V_{GS}}{V_{th}})^2 \\ \Rightarrow |g_m| &\approx 2I_{DSS}(1 - \frac{V_{GS}}{V_{th}}) \propto \sqrt{I_D} I_{DSS}(1 - \frac{V_{GS}}{V_{th}}) \\ \end{split}$$



## High Electron Mobility Transistor - HEMT



# Charge carriers in 2D layer rather than created by dopants

AsGa HEMT nodes & topology



2D electron gas (typ. 100Å)

HEMT technologies (JFET with heterojunction)

- Very high  $e^-$  mobility  $\rightarrow$  high  $g_m$
- High operation frequency up to mm wavelengths
- e<sup>-</sup> conduction spatially separated from donor impurities → no ionized scattering (collisions with impurities)

Allows operation down to sub-Kelvin temperatures (degenerated)

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#### 1/f Noise in a HEMT @ cryo T

Typical decomposition of PSD at T = 51 K: Thermal noise (STh), 1/f noise (S1/f) and one or several G-R noise components (SG-R) :



S. Mouetsi et al, The 1/f Noise in a Two-Dimensional Electron Gas: Temperature and Electric Field Considerations

Noise in semiconductor is affected by various parameters such as conductivity, defect density, temperature, doping concentration, and bias voltage. However, when bias or temperature are varied, the semiconductor properties are no longer constant ... when the temperature is lowered, a shift of the cutoff frequencies towards lower values occurs, new levels appear Nodes & topology



#### Bipolar transistor technologies

Thin semiconductor material common to 2 junctions :

- Homojunctions si/si
  - $\rightarrow$  Bipolar Junction Transistor **BJT**
- Heterostructure III/V as InP/InGaAs or IV/IV as Si/SiGe
  - $\rightarrow$  Heterojunction Bipolar Trans. **HBT**

#### Parameters :

- transconductance
- current gain
- input impedance

$$g_m = \frac{\partial I_C}{\partial V_{BE}}$$
  
$$\beta = \frac{\partial I_C}{\partial I_B}$$
  
$$h_{11} = \frac{\partial V_{BE}}{\partial I_R} = \frac{\beta}{g_m}$$

#### Bipolar access resistances R'

At cryogenic temperatures, weakly doped semiconductor suffer from freeze-out  $\Rightarrow$  increasing of the access resistances



 $R_{BB'}$  and  $R_{EE'}$  access resistances are **combine in a unique R'** 

$$R' = \frac{R_{BB'}}{\beta} + \frac{(\beta+1)R_{EE'}}{\beta}$$

## Transconductance - $g_m$





recombinations, carrier mean free path, thermal decoupling and R'

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_C}{V_T} = \frac{qI_C}{k_B T} \implies g_m \Big|_{T_{cryo}} = \frac{\frac{qI_C}{\eta k_B T_e}}{1 + R' \frac{qI_C}{\eta k_B T_e}}$$

# Current gain $\beta$ and input impedance

Degraded BJT current gain at low temperatures

$$\beta \propto \exp \frac{\Delta E_g}{k_B T}$$
 with  $\Delta E_g = E_{g_E} - E_{g_B} < 0$ 

 $\Delta E_g$ : difference in band gap between the emitter and the base regions and induced by **doping** - **band gap narrowing** 

Example of common commercial transistor : 2N2222 BJT

measured  $\beta$  go from 225 to 35 from room temperature to 77 K  $h_{11} = \frac{\beta}{g_m}$ : Considering  $I_C = 1$ mA  $\rightarrow h_{11}$  (T = 300K) =  $\frac{225}{20}$   $\approx 6 k\Omega$ 

 $h_{11^{(T=77K)}} = \frac{35}{150 \text{ mS}} < 250\Omega \implies \text{fails} \quad Z_{in} > Z_S \text{ at lower temperatures}$ 

#### Heterojunction Bipolar Transistor - HBT

#### Differing semiconductor materials $\Longrightarrow$ Heterojunction

of one, at least, of the junctions of a bipolar transistor

#### $\rightarrow$ high frequency performances

- III-V or IV-IV hetero-junctions are used by using InP/InGaAs or Si/SiGe for instance.
- Si/SiGe is one of the few hetero-junction compatible with standard Si based technology
   SiGe HBT becomes the most popular bipolar technology with competitive speed, and even better, than III-V expensive technologies

VBE

# HBT planar technology



Instru. Cryo. SiGe - PhD D. Prêle

# SiGe and Cryogeny

- ► HBT is usually developed to achieve **high frequencies** perf.
- For Cryogenic applications, alloy of silicon and germanium (SiGe)
  - $\Rightarrow$  Change the  $\beta(\mathbf{T})$

 $\Rightarrow$  Pushes the **freeze-out** at lower temperatures

Si/SiGe heterojunction improve the **emitter injection efficiency**, as compare to BJT, so that it is possible to **increase the base doping**  $\Rightarrow$  **SiGe HBT still work at 4.2 K**, far away temperatures where Si BJT is freezed out

$$\beta_{HBT,SIGe} = \frac{\mu_n L_p N_{dE}}{\mu_p W_B N_{aB}} \exp \frac{\Delta E_{gSI/SIGe} - \Delta E_{gEapp}}{k_B T}$$
$$\beta_{BJT,SI} \approx \frac{\mu_n L_p N_{dE}}{\mu_p W_B N_{aB}} \exp \frac{-\Delta E_{gEapp}}{k_B T}$$

<0 for BJT due to doping

could be >0 due to Ge

 $\beta(T)$  and "band gap *vs* doping"



- ▶ at small *J*<sub>*C*</sub> : recombinations in the base
- ► at large *J*<sub>*C*</sub> : high injection

 $\beta(J_C) \xrightarrow[T]{}$  "bell curve"

<0 for BJT due to doping

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- ▶ at small *J*<sub>*C*</sub> : recombinations in the base
- ► at large *J*<sub>*C*</sub> : high injection

 $\beta(J_C) \xrightarrow[T]{}$  "bell curve"

<0 for BJT due to doping

could be >0 due to Ge

 $\beta(T)$  and "band gap *vs* doping"



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 $\beta(T)$  and "band gap *vs* doping"



#### HBT transconductance

#### As for, BJT, the SiGe HBT transconductance follows

$$g_m = \frac{\frac{q_{I_C}}{\eta k_B T_e}}{1 + R' \frac{q_{I_C}}{\eta k_B T_e}} \quad \text{with} \quad R' = \frac{R_{BB'}}{\beta} + \frac{(\beta + 1)R_{EE'}}{\beta} \quad \text{Brown in } R_{BB'}$$

Below 77 K, the HBT still operates ...

- Strong T<sub>e</sub> decoupling
- ▶ "Start" of freeze-out  $\Rightarrow$  R'  $\nearrow$
- R' effect if  $R' \frac{qI_C}{\eta k_B T_e}$  is comparable or larger than 1
  - ► Large R'
  - Large  $I_C$
  - Low T

#### measured SiGe transconductance



- $T_e \rightarrow$  the 4.2 K measurement fit with qI<sub>C</sub>/k<sub>B</sub>34 K
- R' reduce the measured  $g_m$  (as compare to the ideal law)
- Recombination in the base-emitter depletion region at cryo. T








Gummel plot of a NPN254 SiGe<sub>AMS0.35µm</sub> ( $L_E = 50µm \rightarrow area = 20µm^2$ )





# Calculated $h_{11}$ from $\beta$ and $g_m$ measurements input impedance $h_{11}$

$$h_{11} = \frac{\beta}{g_m}$$

Considering a HBT SiGe with 100  $\mu m^2$  area and  $I_C = 1$  mA

•  $J_C$  is thus equal to 10  $\mu$ A/ $\mu$ m<sup>2</sup> (1 mA/100  $\mu$ m<sup>2</sup>)

Parameters	300 K	77 K	4.2 K
β	180	1400	900
g <sub>m</sub>	30 mS	100 mS	150 mS
h <sub>11</sub>	6 kΩ	$14 \text{ k}\Omega$	6 kΩ































#### **Noise** discussion - on the benefit to have the larger $g_m$

#### **FET noise = THERMAL** noise of the channel resistance

• Output current noise : 
$$S_{i_D} = 4k_BT\frac{2g_m}{3} + K\frac{I_D^{m\geq 2}}{f^{\gamma\approx 1}}$$

• Input voltage noise : 
$$S_{\nu_{GS}} = \frac{S_{i_D}}{g_m^2} = \frac{8k_BT}{3g_m} + \frac{K}{g_m^2} \frac{I_m^{2/2}}{f^{\gamma \approx 1}}$$

#### **Bipolar noise = SHOT** noise of the junctions

• Input voltage noise 
$$S_{\nu_{BE}} = 4k_B T R_{BB'} + \frac{2qI_C}{g_m^2} + K \frac{R_{BB'}I_B^{\alpha \geq 2}}{f^{\gamma \approx 1}}$$
  
 $\approx \frac{4k_B T}{2g_m} + K \frac{R_{BB'}I_B^{\alpha \geq 2}}{f^{\gamma \approx 1}}$ 

• Input current noise 
$$S_{i_B} = 2qI_B + K \frac{I_B^{n \geq 2}}{f^{\gamma \approx 1}}$$

#### SiGe HBT Shot noise and 1/f (?) noise at cryo. T

1/f noise



#### 1/f noise and transistor topology



1/f noise is essentially due to the **non-ideal base current** in bipolar technologies



for MOS its cause come from the **trap on the oxide/channel interface** at the surface of the substrate



JFET channel is geometrically limited only by depleted regions  $\rightarrow$  Less trap than near the surface  $\rightarrow$  Low 1/f noise

+ effect of the size  $\rightarrow 1/f \propto 1/Area$ 

#### Superconducting QUantum Interference Device

SQUID = **Magnetic flux transducer** ⇒ **Voltage** 

The "DC SQUID" is composed of one superconducting ring (Washer) interrupted by two Josephson junctions (x).



Very sensitive magnetometer which combine two physical phenomena :

- Magnetic flux quantization (φ<sub>0</sub> = h/2e ≈ 2.10<sup>-15</sup> W b ou [T.m<sup>2</sup>] ou [V.s]) in a superconducting loop
- 2. Josephson tunneling effect

 $h \approx 6, 6.10^{-34} \, J.s$  or  $[W.s^2]$  Planck constant;  $e \approx 1, 6.10^{-19} \, C$  or [A.s]

#### Magnetic flux quantization in a superconducting ring

Quantum properties of the superconductivity : q = 2e (charge of the Cooper pair) Superconductor is described by a **quantum wave function**  $\psi$ .

In superconducting ring, phase of  $\psi$  continuously change but **must comes** to the same value around a turn  $\rightarrow$  magnetic flux screening can only compensates n magnetic flux quanta  $\phi_0$ :

$$\phi = n\frac{h}{2e} = n\phi_0$$

 $\phi_0 = \frac{h}{2e} \approx 2.10^{-15} Wb$  ou  $[T.m^2]$  ou [V.s] le quantum de flux magnétique

## Josephson junction

2 superconductors separated by a thin  $_{(\approx 10nm)}$  non-superconducting barrier.

#### Josephson tunneling effect :

Cooper pairs of electrons pass through the barrier by tunneling effect, maintaining phase coherence in the process.



Current biasing controls **phase difference**  $\Delta \phi$  **between the two superconductor** according  $I = I_0 sin \Delta \phi$  leading to superconducting **phase modulation**.

#### Interferences

For  $I > I_0 \Rightarrow$  voltage across the junction became >0 Superconducting phase difference evolves with time at the  $\rightarrow$  Josephson frequency:

$$I \approx I_0 \sin \left( 2\pi \frac{V}{\phi_0} t \right) \Rightarrow \frac{f}{V} = \frac{1}{\phi_0} \approx 500 MHz/\mu V$$

SQUID provides at low frequency, average value of interferences.



With no magnetic flux, the 2 junctions oscillate in phase

 $\Rightarrow$  destructive interference.

#### Flux and superconducting phase shift

Magnetic flux leads to an additional phase shift  $2\pi \frac{\phi}{\phi_0}$ 



The two junctions are not in phase for  $\phi \neq n\phi 0$  (periodicity)

#### I(V) and $V(\phi)$ characteristic SQUID (Magnetometer)

- Bias  $< 2I_0$  : no voltage
- ► Bias >  $2I_0$  : SQUID has periodic ( $\phi_0$ ) characteristic V( $\phi$ )



#### SQUID as a trans-impedance amplifier

An input loop is used to convert  $I_{IN}$  in flux  $\phi = \frac{I_{IN}}{M_{IN}}$ :



- Input impedance =  $0 \Omega$
- ▶ Input noise ≈ pA/ $\sqrt{Hz}$
- ► Trans-impedance gain ≈ 100 V/A

# I(V) and $V(\phi)$ overplot

The use of  $V(\phi)$  to mesure I(V)



# A flux feedback to linearize the SQUID characteristic

An other loop is usually used to compensate magnetic flux induced by  $I_{in}$ .

Flux Loked Loop



#### SQUID planar technology



J. S. Bennett et al., Precision Magnetometers for Aerospace Applications: A Review - 2021

#### SQUID gradiometry to remove far field magnetic noise

Pickup consists of 2 integrated rectangular coils connected in series and magnetically coupled to a dc-SQUID in a double parallel washer con?guration via two crossed multiturn input coils



Design and fabrication of multichannel DC SQUIDs for biomagnetic applications S. Yamasaki
# Planar technology and gradiometry



Lawrence Livermore National Laboratory



MEB-ief, SQUID StarCryo

## **SQUID Noise**

- ▶ Junction flicker noise :  $S_V = (\frac{dIc}{dT})^2 (\frac{dV}{dI_C})_I^2 k_B T^2 / (3C_V f)$ ->  $C_V$  the junction heat capacity ->  $\forall$  SQUID geometry random trapping and release of electrons in the junction barrier, which locally decrease the conductivity and cause the *critical current fluctuation*
- Excess 1/f noise observed two orders of magnitude higher

#### **Shunt Johnson noise :** $S_V = 4k_BTR_{\text{shunt}}$

J. Clarke and G. Hawkins, Flicker 1/f noise in Josephson tunnel junctions, Phys. Rev. B 14, 2826 1976

### Magnetic Johnson induced noise from normal metal

Thermal motion of charge carriers in a conducting object causes magnetic field noise can interfere with sensitive detectors :  $\sqrt{S_I} = (4k_b T \mu_0^2 / L_0^2 \rho_n) G \propto \sqrt{\sigma} \rightarrow$  increase with cooling

with  $G = \pi h a^4 / 32 z_0^2$ 



K. Yu et al. Direct Measurement of Thermal Noise and Eddy-current Noise Induced in Metals by Using a ist-order SQUID Gradiometer J. Clem et al. Johnson noise from normal metal near a superconducting SQUID IEEE 1987

## Conclusion

- + Decreasing temperature leads to reduces the thermal noise
- However, shot noise and flicker noise are not directly improved with cooling
- Many examples show significant increase of 1/f noise at cryogenic temperatures

<u>Also</u>: transistor transconductance normalized to a given biasing current is increasing with cooling

However, to reduces the electronic dissipation at cryogenic temperature -> biasing is also reduce leading to not benefiting larger gain at cryo T